



# More gist, better math:

## Fuzzy trace theory-based investigation of the relationship between long-term memory and mathematical skills

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### BACKGROUND

- Working memory and arithmetic fact retrieval performance are predictors of numeracy (Amland et al., 2024, *Psychol. Bull.*). The contribution of LTM to math skills is understudied.
- Fuzzy-trace theory (FTT) differentiates **verbatim** (remembering exact numbers) and **gist** (remembering a general intuition about a number) memory traces and several fine-grained processes associated with them (Reyna & Brainerd, 2023, *Nat. Rev. Psychol.*)
- FTT-driven studies have suggested a positive link between gist and the elementary symbolic magnitude representation (Reyna & Brust-Renck, 2020, *J. Behav. Decis.*) and numeracy (Liberali et al., 2012, *J. Behav. Decis.*).
- The link between gist and verbatim traces and diverse math skills remains understudied.

#### Research questions:

- Is overall performance in the LTM task positively correlated with math skills and math self-concept?
- How do math skills correlate with fine-grained memory processes, including accurate memory recollection (e.g., Gist familiarity for target, and for related stimuli) and faulty memory processes (e.g., Phantom recollection, Erroneous recollection rejection)?
- Are there significant differences in the strengths of the correlations between verbatim and gist processes and diverse math skills?

### TAKE HOME MESSAGES

Overall numerical memory is positively linked to:

- Math reasoning
- Overall arithmetic fluency
- Multiplication and division fluency
- Math self-concept.

No significant associations were found with:

- Addition or subtraction fluency
- Approximate number processing.

While **gist** and **verbatim** processes are associated with math skills, the associations with **gist**-related processes are more prominent.

Even though math self-concept is positively associated with overall numerical memory performance, it is significantly linked only to verbatim-based processes.

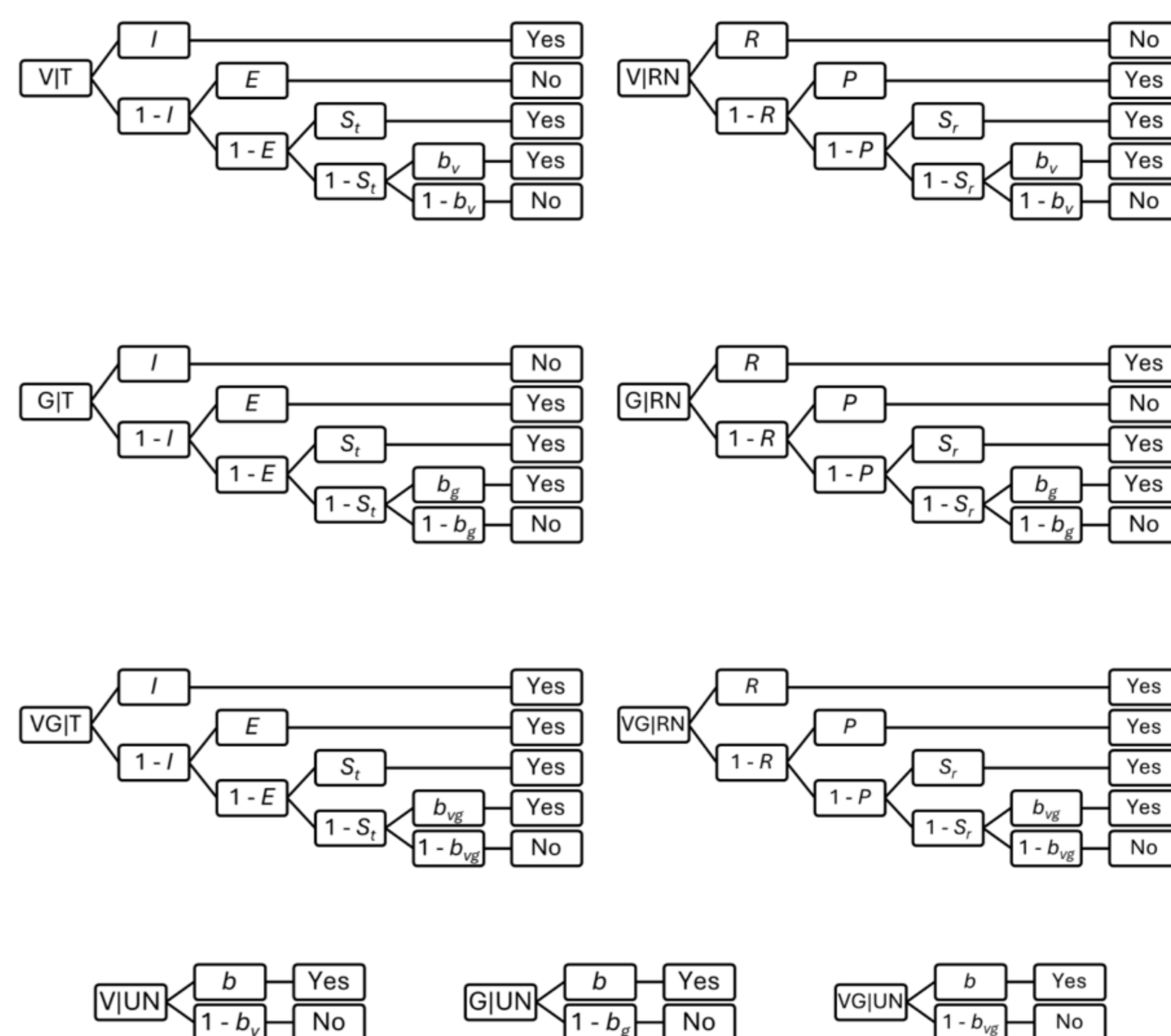
The relationship between long-term memory-related processes and math skills is nuanced, extending beyond a simple positive relationship.

Fuzzy-trace theory and multinomial modeling can provide new paths for numerical cognition research, particularly regarding the relationship between math skills and other memory systems.

### METHOD

- Participants:**  $n = 114$  ( $M = 22.83$ ,  $SD = 3.21$ , range = 18-34 yo)
- Numerical LTM:** a new task based on the conjoint recognition model for a single sample (Brainerd et al., 2010, *JEP:LMC*)
  - Phases: Learning → Retention (a buffer task) → Test
  - 45 sentences (e.g., “The farmer has 7 dogs and 11 horses”)
  - In the test, each is accompanied by one of three questions (Yes / No answer)
    - “Is the stimulus identical to the one from the study phase?” – the correct answer is YES only for target items
    - “Is the stimulus only similar to the one from the study phase?” – the correct answer is YES only for related new items
    - “Is the stimulus either identical to or similar to the one from the study phase?” – correct answer is YES for both target and related new items.
- Tasks for testing math performance:**
  - Approximate Number Sense: Dot comparison task
  - Arithmetic fluency: Math4Speed task
  - Math reasoning: a set of 20 school math problems
  - Math self-concept: Mathematical Ability subscale of SDQ-III

#### Conjoint recognition model



**Processes: Conjoint recognition model parameters:**  $I$  - Identity (retrieving a target's verbatim trace);  $R$  - Recollection rejection (retrieving a verbatim in the case of gist familiarity);  $E$  - Erroneous recollection rejection (retrieving verbatim of similar yet another stimulus);  $P$  - Phantom recollection (retrieving gist of similar yet another stimulus);  $S_t$  - Gist familiarity for the target (retrieving gist when a target is presented);  $S_r$  - Gist familiarity for related stimuli (retrieving gist when a related new item is presented);  $b_v$  - Guessing Yes in “is it target” probe;  $b_g$  - Guessing Yes in “is it new but related” probe;  $b_{vg}$  - Guessing Yes in “either is it target or new related” probe.

### RESULTS

#### Spearman's $\rho$ correlations of memory and math skills

|                | LTM    | ANS    | M4S (+) | M4S (-) | M4S (x) | M4S (÷) | M4S All | M reasoning |
|----------------|--------|--------|---------|---------|---------|---------|---------|-------------|
| Numerical LTM  | -      | -      | -       | -       | -       | -       | -       | -           |
| ANS            | 0.08   | -      | -       | -       | -       | -       | -       | -           |
| M4S (+)        | 0.14   | 0.12   | -       | -       | -       | -       | -       | -           |
| M4S (-)        | 0.11   | 0.08   | 0.82*** | -       | -       | -       | -       | -           |
| M4S (x)        | 0.20*  | 0.10   | 0.72*** | 0.71*** | -       | -       | -       | -           |
| M4S (÷)        | 0.23*  | -0.00  | 0.59*** | 0.69*** | 0.73*** | -       | -       | -           |
| M4S All        | 0.20*  | 0.08   | 0.85*** | 0.89*** | 0.91*** | 0.88*** | -       | -           |
| M reasoning    | 0.25** | 0.29** | 0.51*** | 0.51*** | 0.50*** | 0.51*** | 0.57*** | -           |
| M self-concept | 0.17†  | -0.03  | 0.48*** | 0.55*** | 0.47*** | 0.55*** | 0.58*** | 0.59***     |

\* $p < .05$ , two-tailed. \*\* $p < .01$ , two-tailed. \*\*\* $p < .001$ , two-tailed. † $p < .05$ , one-tailed

#### Latent-trait model analysis (R's treeBUGS) & estimated parameters of conjoint recognition model

|          | Mean | SE   | 95% CL      |
|----------|------|------|-------------|
| $b_g$    | 0.29 | 0.02 | 0.24 – 0.34 |
| $b_v$    | 0.22 | 0.02 | 0.18 – 0.27 |
| $b_{vg}$ | 0.38 | 0.03 | 0.33 – 0.42 |
| $E$      | 0.13 | 0.04 | 0.05 – 0.21 |
| $S_r$    | 0.50 | 0.07 | 0.34 – 0.63 |
| $S_t$    | 0.84 | 0.04 | 0.76 – 0.91 |
| $P$      | 0.04 | 0.04 | .001 – 0.13 |
| $R$      | 0.47 | 0.04 | 0.39 – 0.54 |
| $I$      | 0.04 | 0.02 | .004 – 0.09 |

- Model fitting: 50000 iterations of MCMC sampler and a 5000 burn-in period.
- Goodness of fit:
  - T1: Observed = 0.07, Predicted = 0.07,  $p = 0.514$
  - T2: Observed = 4.03, Predicted = 3.54,  $p = 0.359$

#### Spearman's $\rho$ for model parameters and math skills

|          | ANS   | M4S (+) | M4S (-) | M4S (x) | M4S (÷) | M4S All | M reasoning | M self-concept |
|----------|-------|---------|---------|---------|---------|---------|-------------|----------------|
| $b_v$    | -0.09 | -0.11   | -0.12   | -0.20*  | -0.17†  | -0.18†  | -0.22*      | -0.14          |
| $b_g$    | -0.15 | -0.18†  | -0.17†  | -0.25** | -0.23*  | -0.24** | -0.30***    | -0.15          |
| $b_{vg}$ | -0.10 | -0.16†  | -0.16†  | -0.24*  | -0.21*  | -0.22*  | -0.29**     | -0.18†         |
| $E$      | 0.13  | 0.07    | 0.13    | 0.08    | 0.11    | 0.11    | 0.26***     | 0.23**         |
| $S_r$    | 0.12  | 0.15    | 0.13    | 0.25**  | 0.18†   | 0.21*   | 0.32***     | 0.15           |
| $S_t$    | 0.15  | 0.16†   | 0.11    | 0.23*   | 0.17†   | 0.19*   | 0.28**      | 0.15           |
| $P$      | -0.06 | -0.13   | -0.12   | -0.14   | -0.16†  | -0.16†  | -0.18†      | -0.13          |
| $R$      | 0.19* | 0.07    | 0.07    | 0.13    | 0.19*   | 0.14    | 0.27**      | 0.19*          |
| $I$      | -0.03 | 0.17†   | 0.13    | 0.18†   | 0.22*   | 0.20*   | 0.22*       | 0.21*          |